

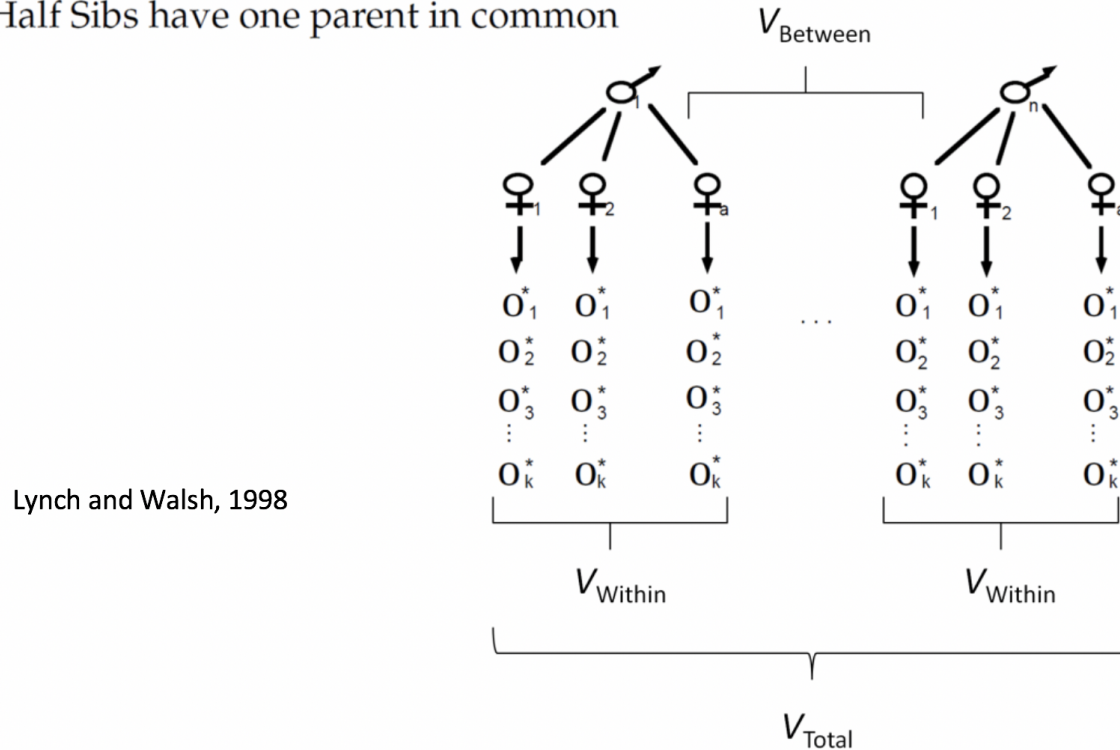


Resemblance between relatives

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Oct. 22nd, 2018

Half Sibs

Half Sibs have one parent in common



Lynch and Walsh, 1998

With an experimental design including family structure, we can partition phenotypic variance into that **among families** and that **within families**.

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The covariance between two individuals in the same group (y_{ij} and y_{ik}) is

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As similarity within families increases, variation among families increases.

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Thus, if we have the variance partitioned into the between families (σ_B^2) and within families (σ_W^2), and these sum to the phenotypic variance ($\sigma_P^2 = \sigma_B^2 + \sigma_W^2$), then the intraclass correlation is:

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Assumptions

1. Diploidy
2. Autosomal loci
3. Linkage equilibrium
4. No maternal effects
5. No GxE interactions
6. No selection

General framework for calculating resemblance

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The more closely related a group of individual is, the greater the probability they share alleles that are IBD, and therefore the greater the covariance.

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If allele x_i carried by individual X is IBD to allele y_k in Y, then the covariance due to this allele is:

$$\begin{aligned} \text{Cov}(\alpha_i, \alpha_k) &= E[(\alpha_i - \mu_\alpha)(\alpha_k - \mu_\alpha)] \\ &= E[(\alpha_i - \mu_\alpha)^2] \\ &= \sigma_\alpha^2 \end{aligned}$$

Because $\alpha_i = \alpha_k$ if alleles x_i and y_k are IBD.

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$$\begin{aligned} Cov_{\alpha}(X, Y) &= P(x_i \equiv y_k)Cov(\alpha_i, \alpha_k) + P(x_i \equiv y_l)Cov(\alpha_i, \alpha_l) \\ &\quad + P(x_j \equiv y_k)Cov(\alpha_j, \alpha_k) + P(x_j \equiv y_l)Cov(\alpha_j, \alpha_l) \\ &= 4f_{XY}\sigma_{\alpha}^2 \\ &= 2f_{XY}\sigma_A^2 \end{aligned}$$

Because $\sigma_A^2 = \sigma_{\alpha_i}^2 + \sigma_{\alpha_j}^2 = 2\sigma_{\alpha}^2$ and $\alpha_i = \alpha_k$ when alleles i and k are IBD.

Parent-offspring

Recall that the coefficient of co-ancestry between a non-inbred parent and non-inbred offspring is $1/4$. Thus, the coefficient for σ_A^2 is $1/2$.

Consider the covariance between parent (P) and offspring (O), if we assume the parents are unrelated, then all offspring can't share a common dominance deviation ($\sigma_D^2 = 0$). Therefore,

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$$\begin{aligned} Cov(P, O) &= Cov(A + D, 1/2A) \\ &= 1/2Cov(A, A) + 1/2Cov(A, D) = \frac{1}{2}\sigma_A^2 \end{aligned}$$

Because $Cov(A, D) = 0$ from Ch.8.