Genetic components of variance

Jinliang Yang
Oct. 22th, 2018

Announcements

Second Exam

- The American Society of Agronomy meetings: Nov. 4-7th, 2018
- Exam Date Nov. 9 or Nov. 12?
- Will post 2017 exam and key for your preparation.

Announcements

Second Exam

- The American Society of Agronomy meetings: Nov. 4-7th, 2018
- Exam Date Nov. 9 or Nov. 12?
- Will post 2017 exam and key for your preparation.

Homework

- Due next **Wednesday (Oct. 31th, 2018) at 8:00AM**

Phenotypic variance partitioning

Phenotypic model for single locus: P = A + D + E

$$\sigma_G^2 = \sigma_A^2 + \sigma_D^2 + \sigma_E^2$$

Phenotypic variance partitioning

Phenotypic model for single locus: P = A + D + E

$$\sigma^2_G = \sigma^2_A + \sigma^2_D + \sigma^2_E$$

Genotypic effect model for multiple loci

$$G_{ijkl} = \mu + (\alpha_i + \alpha_j + \delta_{ij}) + (\alpha_k + \alpha_l + \delta_{kl}) + I_{ijkl}$$

Phenotypic variance partitioning

Phenotypic model for single locus: P = A + D + E

$$\sigma^2_G = \sigma^2_A + \sigma^2_D + \sigma^2_E$$

Genotypic effect model for multiple loci

$$G_{ijkl} = \mu + (\alpha_i + \alpha_j + \delta_{ij}) + (\alpha_k + \alpha_l + \delta_{kl}) + I_{ijkl}$$

Genotypic variance:

$$\begin{aligned}
\sigma^2_{G_{ijkl}} =& (\sigma^2_{\alpha_i} + \sigma^2_{\alpha_j} + \sigma^2_{\delta_{ij}}) \\
& + (\sigma^2_{\alpha_k} + \sigma^2_{\alpha_l} + \sigma^2_{\delta_{kl}}) + \sigma^2_{I_{ijkl}} \\
=& (\sigma^2_{\alpha_i} + \sigma^2_{\alpha_j} + \sigma^2_{\alpha_k} + \sigma^2_{\alpha_l}) \\
& + (\sigma^2_{\delta_{ij}} + \sigma^2_{\delta_{kl}}) + \sigma^2_{I_{ijkl}} \\
=& \sigma^2_A + \sigma^2_D + \sigma^2_I
\end{aligned}$$

Additive and dominance variance

Genotype	Freq	Breeding Value	σ_A^2	Dominance Deviation	σ_D^2
A_1A_1	p^2	$2q\alpha$	$(2q\alpha)^2$	$-2q^2d$	$(-2q^2d)^2$
A_1A_2	$2pq$	$(q-p)\alpha$	$(q-p)^2\alpha^2$	$2pqd$	$(2pqd)^2$
A_2A_2	q^2	$-2p\alpha$	$(-2p\alpha)^2$	$-2p^2d$	$(-2p^2d)^2$

Additive and dominance variance

Genotype	Freq	Breeding Value	σ_A^2	Dominance Deviation	σ_D^2
A_1A_1	p^2	$2q\alpha$	$(2q\alpha)^2$	$-2q^2d$	$(-2q^2d)^2$
A_1A_2	$2pq$	$(q-p)\alpha$	$(q-p)^2\alpha^2$	$2pqd$	$(2pqd)^2$
A_2A_2	q^2	$-2p\alpha$	$(-2p\alpha)^2$	$-2p^2d$	$(-2p^2d)^2$

According to the definition of variance:

$$\begin{aligned} Var(X) &= \sigma_X^2 \\ &= E[X^2] - E[X]^2 \end{aligned}$$

And the means of **breeding value** and **dominance deviation** are **0**.

Additive and dominance variance

Genotype	Freq	Breeding Value	σ_A^2	Dominance Deviation	σ_D^2
A_1A_1	p^2	$2q\alpha$	$(2q\alpha)^2$	$-2q^2d$	$(-2q^2d)^2$
A_1A_2	$2pq$	$(q-p)\alpha$	$(q-p)^2\alpha^2$	$2pqd$	$(2pqd)^2$
A_2A_2	q^2	$-2p\alpha$	$(-2p\alpha)^2$	$-2p^2d$	$(-2p^2d)^2$

According to the definition of variance:

$$\begin{aligned} Var(X) &= \sigma_X^2 \\ &= E[X^2] - E[X]^2 \end{aligned}$$

And the means of **breeding value** and **dominance deviation** are **0**.

The additive genetic variance **in a HWE population** is:

$$\begin{aligned} \sigma_A^2 &= 2pq\alpha^2 \\ &= 2pq(a + d(q-p))^2 \\ \sigma_D^2 &= (2pqd)^2 \end{aligned}$$

Additive and dominance variance

Genotype	Freq	Breeding Value	σ_A^2	Dominance Deviation	σ_D^2
A_1A_1	p^2	$2q\alpha$	$(2q\alpha)^2$	$-2q^2d$	$(-2q^2d)^2$
A_1A_2	$2pq$	$(q-p)\alpha$	$(q-p)^2\alpha^2$	$2pqd$	$(2pqd)^2$
A_2A_2	q^2	$-2p\alpha$	$(-2p\alpha)^2$	$-2p^2d$	$(-2p^2d)^2$

Additive and dominance variance

Genotype	Freq	Breeding Value	σ^2_A	Dominance Deviation	σ^2_D
A_1A_1	p^2	$2q\alpha$	$(2q\alpha)^2$	$-2q^2d$	$(-2q^2d)^2$
A_1A_2	$2pq$	$(q-p)\alpha$	$(q-p)^2\alpha^2$	$2pqd$	$(2pqd)^2$
A_2A_2	q^2	$-2p\alpha$	$(-2p\alpha)^2$	$-2p^2d$	$(-2p^2d)^2$

Cov(A, D)?

$$\begin{aligned} Cov(X,Y) &= E([X-E(X)][Y-E(Y)]) \\ &= E(XY) - E(X)E(Y) \end{aligned}$$

where,

$$E(XY) = \sum_i \sum_j x_i y_j Pr(X = x_i, Y = y_j)$$

Repeatability of measurements

The repeatability is the proportion of total variance of single measurements that is due to permanent differences between individuals.

$$r = \frac{\sigma_G^2 + \sigma_{Eg}^2}{\sigma_P^2}$$

Repeatability of measurements

The repeatability is the proportion of total variance of single measurements that is due to permanent differences between individuals.

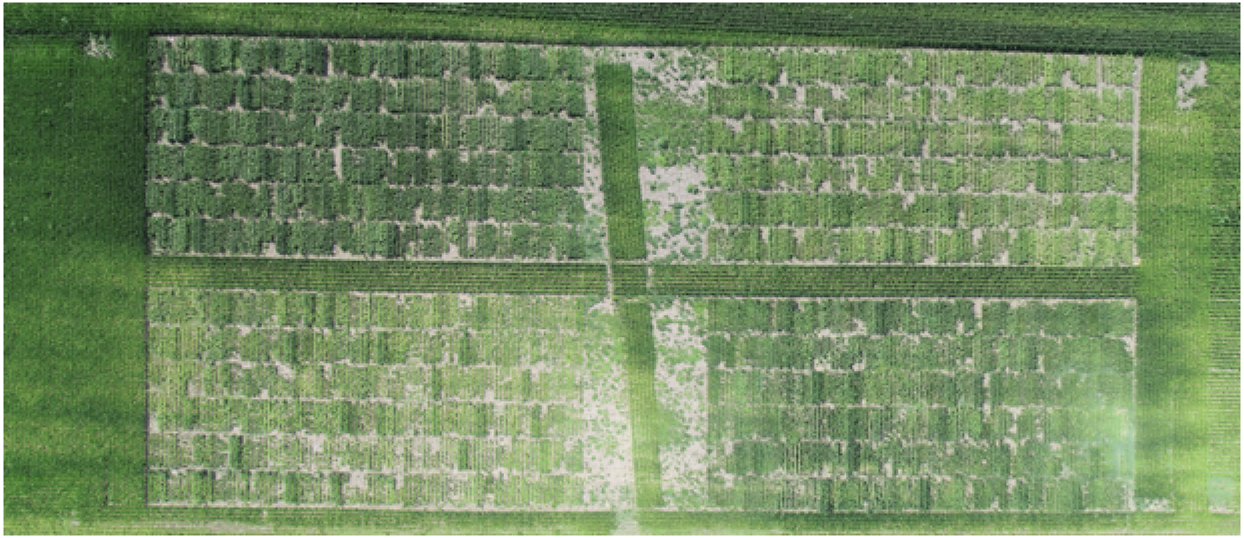
$$r = \frac{\sigma^2_G + \sigma^2_{Eg}}{\sigma^2_P}$$

Two types of non-genetic variance

1. **Special environmental variance**: within individual variation arising from temporary or localized circumstances.
2. **General environmental variance**: non-genetic variance contributing to the between-individual variation that is permanent.

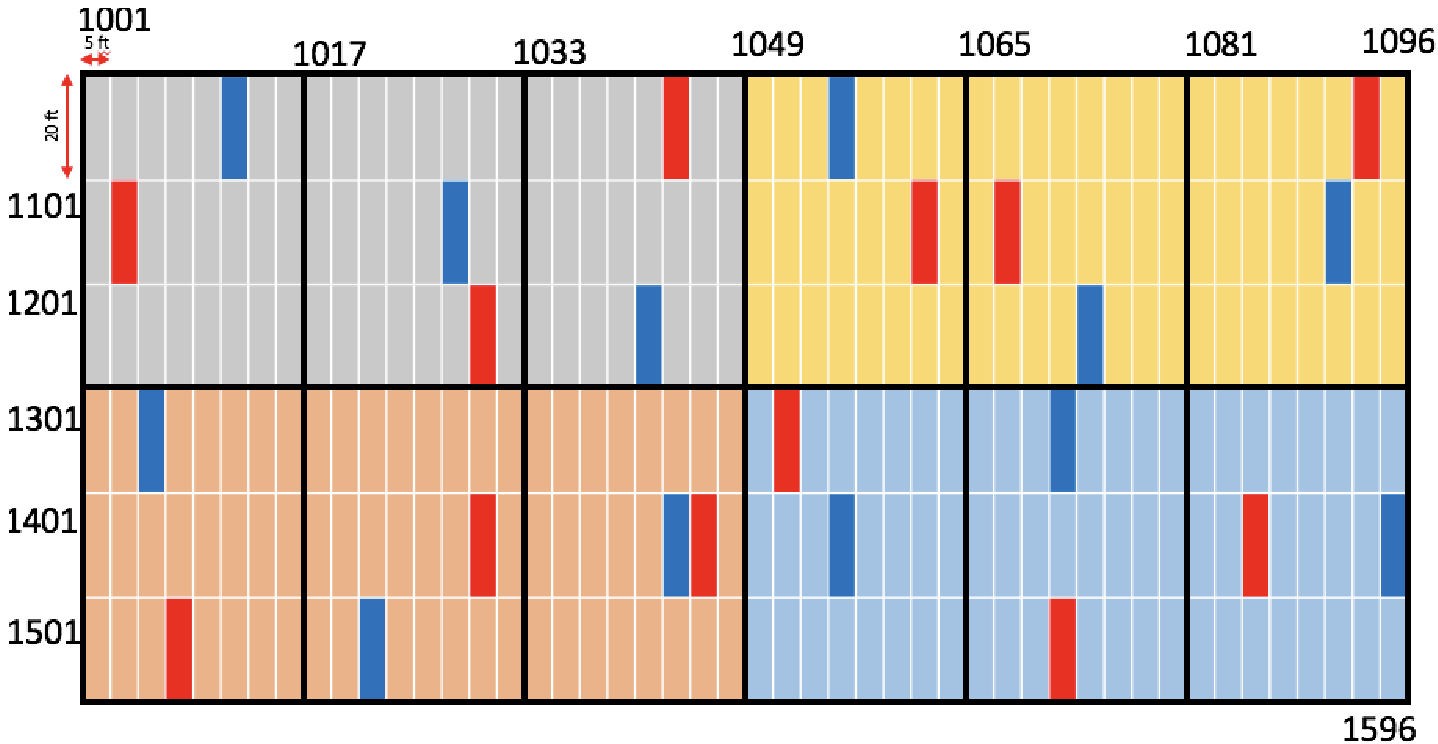
How to get a measurement with precision?

- A set of 300 pure inbred maize lines
- +N and -N treatments, with two replications each
- 30 people to dig out roots and to collect rhizosphere microbiome



How to get a measurement with precision?

1. Randomization
2. Blocking
3. Check plants
4. Replication



Repeatability of measurements

One way to increase the accuracy of measurements is to use the average of multiple measurements instead of just one.

$$r_n = \frac{\sigma_G^2 + \sigma_{Eg}^2}{\sigma_{P(n)}^2}$$

$$\sigma_{P(n)}^2 = \sigma_G^2 + \sigma_{Eg}^2 + \frac{1}{n}\sigma_{Es}^2$$

Repeatability of measurements

One way to increase the accuracy of measurements is to use the average of multiple measurements instead of just one.

$$r_n = \frac{\sigma_G^2 + \sigma_{Eg}^2}{\sigma_{P(n)}^2}$$

$$\sigma_{P(n)}^2 = \sigma_G^2 + \sigma_{Eg}^2 + \frac{1}{n}\sigma_{Es}^2$$

The repeatability of the average of multiple measurements is:

$$\begin{aligned} r_n &= \frac{\sigma_G^2 + \sigma_{Eg}^2}{\sigma_{P(n)}^2} \\ &= 1 - \frac{\frac{1}{n}\sigma_{Es}^2}{\sigma_{P(n)}^2} \end{aligned}$$

If repeatability is low, taking multiple measurements and avaraging them can really improve accuracy.